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CORRIGENDA

Yue-Sheng Wang and Duo Wang, Scattering of elastic waves by a rigid cylindrical inclusion partially debonded from its surrounding matrix—I. SH case. *Int. J. Solids Structures*, Vol. 33, No. 19, pp. 2789–2815, 1996.
Yue-Sheng Wang and Duo Wang, Scattering of elastic waves by a rigid cylindrical inclusion partially debonded from its surrounding matrix—II. P and SV cases. *Int. J. Solids Structures*, Vol. 33, No. 19, pp. 2816–2840, 1996.

In Part I of the above referenced series paper, eqn (56) is incorrect when $n > 1$. It follows from the book by Muskhelishvili (1953) that the general solution of eqn (55) should be

$$\phi_k(\theta) = \frac{\mathrm{i}\tau_0}{\mu_0}\frac{X(\theta)}{\pi \mathrm{i}}\sum_{l=1}^{n}\int_{a_l}^{b_l}\cos(\zeta-\theta_0)X^{-1}(\zeta)\left[\cot\left(\frac{\zeta-\theta}{2}\right)-\tan\frac{\theta}{2}\right]\mathrm{d}\zeta + X(\theta)P_{n-1}\left(\tan\frac{\theta}{2}\right)$$

where $X(\theta)$ is given by eqn (57) and $P_{n-1}(\ )$ is a polynomial of order $n-1$ in terms of $\tan(\theta/2)$. The unknown coefficients of $P_{n-1}(\ )$ may be determined by the single-valued condition (31). For the case of $n = 1$, $P_{n-1}(\ )$ is a constant P_0. Using eqn (31), one may have $P_0 = 0$, and then arrive at eqn (56). That is to say, eqn (56) is correct only when $n = 1$. Since only the case of one debond was considered in detail in that paper, no errors are included in other equations.

It is also noted that, in Part II, the footnote indicating the change of author's address should be marked on the first author, Yue-Sheng Wang.

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Muskhelishvili, N. I. (1953). *Singular Integral Equations*. Noordhoff, Leyden.

T. R. Nordenholz and O. M. O'Reilly, On steady motions of an elastic rod with application to contact problems. *Int. J. Solids Structures*, Vol. 34, No. 9, pp. 1123–1143, 1997.

The purpose of this corrigendum is to correct some statements concerning the invariance requirements for constrained theories that we assumed in our paper. These corrections do not apply when dealing with an unconstrained theory. In our paper, it was assumed that $\mathbf{n}$, $\mathbf{m}^\alpha$ and $\mathbf{k}^\alpha$ were objective [cf. eqn (12)]. This leads to the conclusions that the indeterminate functions p_s are invariant under superposed rigid body motions, and that $\dot{\gamma}$, $\mathbf{F}$, $\mathbf{M}$ and $\mathbf{L}^\alpha$ are objective [cf. eqns (13) and (14)].

As usual, it is assumed that the forces $\mathbf{n}$, $\mathbf{m}^\alpha$ and $\mathbf{k}^\alpha$ can be additively decomposed:

$$\mathbf{n} = \hat{\mathbf{n}}+\bar{\mathbf{n}}, \quad \mathbf{k}^\alpha = \hat{\mathbf{k}}^\alpha+\bar{\mathbf{k}}^\alpha, \quad \mathbf{m}^\alpha = \hat{\mathbf{m}}^\alpha+\bar{\mathbf{m}}^\alpha, \tag{C.1}$$

where the overbar and the hat denote the constraint and determinate responses, respectively. Then, following Casey and Carroll (1996), and O'Reilly and Turcotte (1996), the correct invariance requirements are to assume that only $\hat{\mathbf{n}}$, $\hat{\mathbf{k}}^\alpha$ and $\hat{\mathbf{m}}^\alpha$ are objective:

$$\hat{\mathbf{n}}^{+} = \mathbf{Q}\hat{\mathbf{n}}, \quad (\hat{\mathbf{k}}^{\alpha})^{+} = \mathbf{Q}\hat{\mathbf{k}}^{\alpha}, \quad (\hat{\mathbf{m}}^{\alpha})^{+} = \mathbf{Q}\hat{\mathbf{m}}^{\alpha}. \tag{C.2}$$

For constrained theories, (C.2) does not imply eqns (13) and (14). In addition, the properly invariant theory needs to be modified in a manner that is easily inferred from Section 4 of O'Reilly and Turcotte (1996). However, for an unconstrained theory, (C.2) is identical to eqn (12), and eqn (14) also holds. Finally, as an unconstrained theory was used in the examples discussed in our paper, the corrections reported here have no effect on them.

Also, eqn (9) should read

$$\mathbf{n} = \frac{\partial \Phi}{\partial \gamma_{3k}} \mathbf{d}_k + \frac{\partial \Phi}{\partial \kappa_{\alpha 3}} \mathbf{d}'_{\alpha}, \quad \mathbf{m}^{\alpha} = \frac{\partial \Phi}{\partial \kappa_{\alpha k}} \mathbf{d}_k, \quad \mathbf{k}^{\alpha} = \frac{\partial \Phi}{\partial \gamma_{\alpha k}} + \frac{\partial \Phi}{\partial \kappa_{\beta \alpha}} \mathbf{d}'_{\beta}$$

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Casey, J. and Carroll, M. M. (1996). Discussion of "A treatment of internally constrained materials," by J. Casey. *ASME Journal of Applied Mechanics* **63**, 240.

O'Reilly, O. M. and Turcotte, J. S. (1996). Some remarks on invariance requirements for constrained rods. *Mathematics and Mechanics of Solids* **1**, 343–348.

A. E. Giannakopoulos and S. Suresh, Indentation of solids with gradients in elastic properties: Part I. Point force. *Int. J. Solids Structures*, Vol. 34, No. 19, pp. 2357–2392, 1997; and A. E. Giannakopoulos and S. Suresh, Indentation of solids with gradients in elastic properties: Part II. Axisymmetric indenters. *Int. J. Solids Structures*, Vol. 34, No. 19, pp. 2393–2428, 1997.

A reference to L. N. Ter-Mkrtich'ian, Some problems in the theory of elasticity of nonhomogeneous elastic media. *PMM*, Vol. 25, pp. 1120–1125, 1961, which is from the Russian Literature, was inadvertently left out from the above two-part paper published recently in this journal. This was brought to our attention by Prof. Bertil Storåkers of the Royal Institute of Technology, Stockholm, Sweden, to whom we are grateful. In this paper by Ter-Mkrtich'ian, mathematical formulations for the axisymmetric indentation of an elastic half-space with exponential variations in Young's modulus normal to the indented surface were treated, by recourse to Love's displacement potential and Sneddon's Hankel transform (see our two-part paper for appropriate references for these methods). The theoretical results for the exponential model reported in the initial steps of the derivation in Part I of our two-part paper match the earlier results of Ter-Mkrtich'ian. Full solutions for the particular case of a point force and, the surface vertical displacement, $w(r)$, were not presented by Ter-Mkrtich'ian; these can be found in Part I. The stresses were also formulated in Ter-Mkrtich'ian (1961) in a manner which was not amenable for direct quantification. Equation (2.16) of Ter-Mkrtich'ian (1961) apparently contains an error; the correct form is presented in eqn (41) of Part I.

We have reported, in our aforementioned two-part paper, explicit analytical solutions for the force-indenter penetration and force-contact radius relations, as well as for the stress fields for the point force and axisymmetric indenters, in such a way that the predictions can be compared directly with experimental observations. In addition, new models for power-law spatial variation in Young's modulus and complete finite-element analyses for both the exponential and power-law models were presented in our two-part paper.